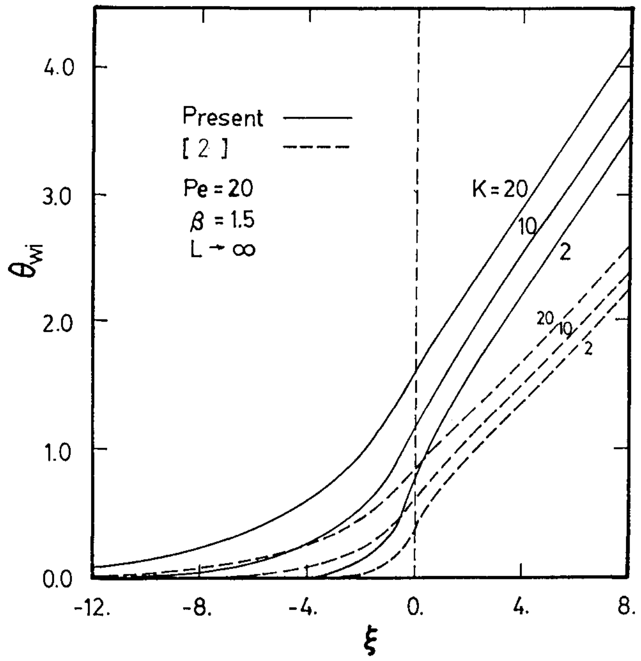

Fig. 2 Comparison of the interfacial temperatures in the neighborhood of the beginning end of the heated section.

laminar forced convection over a flat plate by the integral method.[9]

According to the present investigation, it can be concluded that radial wall conduction becomes important for a system with a high wall-to-fluid thermal conductivity ratio K, thick pipe wall (β much greater than unity), high Peclet number Pe, and short heating length L.

Acknowledgment

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Evaporation from a Drop Suspended in an Electric Field

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Introduction

SINGLE droplet evaporation has been a subject of intensive investigation for years, but the effects of an electric field upon the evaporation rates have received very little attention. In recent years there has been a renewed interest in the applications of an electric field to promote the heat and mass transfer processes. This favorable enhancement is due to the induced motion inside and outside the drop caused by the charge built up in the neighborbood of the drop surface, which leads to the formation of electrical stresses. The electrically generated motion was first observed and calculated by Taylor[1] and later was employed in the studies of heat and mass transfer by various investigators. Morrison[2] solved the conjugate heat and mass transfer problem at high Peclet numbers, in which he found that the transfer coefficient is proportional to the field strength. At low Peclet numbers, the dominant region of heat and mass transport is not thin enough for boundary layer approximations to be valid; hence, the analysis of the transfer processes has to be started with the convective diffusion equation. Griffith and Morrison[3] expanded the dependent quantities in powers of Peclet number, and then followed the usual perturbation procedure to obtain the temperature distribution for systems with transfer resistance in the continuous phase. In the high Peclet number range, Sharper and Morrison[4] presented numerical solutions to external problems using the finite-difference numerical method.

As a droplet is moving slowly under the influences of gravity and an electric field, a translational motion is superimposed on the Taylor flowfield, and this results in much more complicated flow patterns. Chang et al.[5] and Chang and Berg[6] derived the hydrodynamical solutions for drops translating at low and intermediate Reynolds numbers, respectively. They then incorporated this information to study the heat and mass transfer for both the inside and the outside of the fluid sphere, but their results are strictly applicable at high Peclet numbers. Chung et al.[7] examined the convective diffusion equation for an internal heat transfer problem by an alternating direction implicit method.

The enhancement of heat and mass transfer due to an electric field is a good indication that the rate of evaporation also would be increased because the mass of liquid lost by evaporation is a linear function of the transfer coefficient. The purpose of this Note, therefore, is to extend the work of Morrison[2] to investigate the evaporation from a spherical fluid droplet suspended in an electric field.

Mathematical Model

In this section we shall formulate a model for a pure liquid drop with initial radius R_0 and temperature T_0 suspended in a hot quiescent environment at temperature T_∞, and with the two-phase system subjected to a uniform electric field of strength E. With the physical system so described, we are now

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restricting ourselves to the following assumptions: 1) The droplet remains spherical at all times (an excellent approximation for tiny drops); 2) the fluids possess Newtonian behaviors with constant physical properties and the electrical conductivities of the two fluids are low enough for the magnetic field to be negligible; 3) the electrically induced motion is assumed to be axisymmetric and is in the creeping regime; 4) thin thermal and concentration boundary layers exist on each side of the liquid surface; and 5) the gas-phase processes and the motion inside the drop are quasisteady, however transience is retained for liquid heating. With these simplifications, the governing equations for these processes may be written, in dimensionless form, as follows:

$$U_{r_1}\frac{\partial Z_1}{\partial \xi_1} + U_{\theta_1}\frac{\partial Z_1}{\partial \theta} = -\frac{1}{Pe}\frac{\partial^2 Z_1}{\partial \xi_1^2} \tag{1}$$

$$U_{r_1}\frac{\partial X}{\partial \xi_1} + U_{\theta_1}\frac{\partial X}{\partial \theta} = -\frac{1}{Pe}\left(\frac{D}{\alpha_1}\right)\frac{\partial^2 X}{\partial \xi_1^2} \tag{2}$$

for the gas phase, and

$$\frac{\partial Z_2}{\partial \tau} - Pe\left[U_{r_2}\frac{\partial Z_2}{\partial \xi_2} + U_{\theta_2}\frac{\partial Z_2}{\partial \theta}\right] = \frac{\partial^2 Z_2}{\partial \xi_2^2} \tag{3}$$

for the liquid phase.

In the above equations, the velocity is taken to be negative as dictated by the conductivity ratio of the liquid and the gas, which usually has a value much greater than unity. The geometry of the flowfield, the coordinate system, and the nomenclature closely follow that of Morrison,[2] except for a minor notational difference. Here, the coordinate ξ_i is the normal distance from the interface weighted with $\sqrt{\alpha_1/\alpha_i}/R$. The dependent variable X is the mass fraction of the vaporizing species in the gas phase. Because of the moving nature of the boundary, the dimensionless time τ is defined as $[\int_0^t (\alpha_1/R^2)\,\mathrm{d}t]$. The Peclet number in this context is based on the thermal diffusivity of the continuous phase, i.e., $Pe = (2|U_{c\tau}|R/\alpha_1)$ where $U_{c\tau}$ is the maximum speed generated by the electric field. For further details, the readers are referred to the classic paper by Taylor.[1] To complete the formulation, Eqs. (1–3) are solved with the following boundary conditions: $Z_1(\tau,\theta,\infty)=0$, $X(\tau,\theta,\infty)=0$, $Z_2(\tau,\theta,-\infty)=1$; $Z_1(\tau,\theta,0)=Z_s(\tau)$, $X(\tau,\theta,0)=X_s(\tau)$, $Z_2(\tau,\theta,0)=Z_s(\tau)$; and $Z_2(0,\theta,\xi_2)=1$.

Two distinct differences between the above equations and Morrison's[2] are the time-dependent interfacial boundary condition, and the coupling between the interfacial temperature and concentration through a phase change relation to accommodate the evaporation. Even with the added complications, the solution to the present problem can be obtained from the corresponding time-independent boundary problem by using Duhamel's theorem.

Method of Solution

To solve Eqs. (1–3) subjected to the constraints specified, we shall first attack the problem with the interfacial temperature T_s and mass fraction X_s treated as constants. The removal of transience in the boundary condition makes it possible for a similarity analysis to be applicable. As done by Morrison,[2] the governing equations are rewritten in terms of new variables defined as:

$$\eta_i = \sqrt{Pe}\,\xi_i \sin^2\theta \cos\theta \tag{4a}$$

$$\zeta(\tau,\theta) = -\frac{\sin^4\theta}{4} + \frac{\tan^4\theta e^{4Pe\tau}}{4(1+\tan^2\theta e^{2Pe\tau})^2} \tag{4b}$$

For the continuous phase, the second term of Eq. (4b) is not needed because a quasisteady assumption had been invoked. The introduction of Eqs. (4) transforms the governing equations to the standard form of the one-dimensional diffusion equation with ϕ as the transport coefficient,

$$\frac{\partial Z}{\partial \zeta} = \phi\frac{\partial^2 Z}{\partial \eta^2} \tag{5}$$

where Z stands for X, Z_1, and Z_2, respectively. It should be noted that the coefficient ϕ equals (D/α_1) when $Z = X$, and is unity otherwise. In the same manner, the constraints become: $Z_1(\zeta,\infty)=0$, $Z_2(\zeta,-\infty)=1$, $X(\zeta,\infty)=0$; $Z_1(\zeta,0)=Z_s$, $Z_2(\zeta,0)=Z_s$, $X(\zeta,0)=X_s$; and $Z_1(0,\eta_1)=0$, $Z_2(0,\eta_2)=1$, $X(0,\eta_1)=0$.

The solution to Eq. (5) when X_s and Z_s are constant is represented by the well-known error function. Once the solution to the time-independent boundary condition is known, Duhamel's principle can be applied directly to yield the results

$$Z_1(\zeta,\eta_1) = Z_s\left\{1-\operatorname{erf}\left[\frac{\eta_1}{2\sqrt{\zeta_{ss}(\theta)}}\right]\right\} \tag{6}$$

$$Z_2(\zeta,\eta_2) = Z_s - \frac{\partial}{\partial \tau}\int_0^\tau [1-Z_s(\tau^*)]\operatorname{erf}\left[\frac{\eta_2}{2\sqrt{\zeta(\theta,\tau-\tau^*)}}\right]\mathrm{d}\tau^* \tag{7}$$

$$X(\zeta,\eta_1) = Z_s\left[1-\operatorname{erf}\left\{\frac{\eta_1}{2}\sqrt{(\alpha_1/D)/[\zeta_{ss}(\theta)]}\right\}\right] \tag{8}$$

These solutions are incomplete because the parameters Z_s and X_s are still to be determined. For a single-component drop, the Cox-Antoine equation correlates the vapor pressure better than the Clausius-Clapeyron equation, however, the disagreement seems to disappear for small drops on the order of 100 μm as demonstrated by Hallett.[8] Due to the convenient form of the Clausius-Clapeyron equation, it is used here to relate the interfacial parameters

$$\frac{1}{X_s} = 1 - \frac{W_1}{W_2}\left\{1 - P_\infty \exp\left[-\frac{\Delta H_v}{R_g T_b}\left(1-\frac{T_b}{T_s}\right)\right]\right\} \tag{9}$$

where ΔH_v is the latent heat of vaporization assumed to be temperature-independent, R_g the specific gas constant, W the molecular weight, and T_b the normal boiling temperature of the drop. The local energy balance at the contacting surface provides an additional relation between X_s and Z_s

$$\frac{X_s}{\sqrt{\zeta_{ss}}} = \frac{K_1(T_0-T_\infty)}{\rho_1 \Delta H_v \sqrt{D\alpha_2}}\left[\frac{K_2}{K_1}\frac{\partial}{\partial \tau}\int_0^\tau \frac{1-Z_s(\tau^*)}{\sqrt{\zeta(\theta,\tau-\tau^*)}}\,\mathrm{d}\tau^* - \sqrt{\frac{\alpha_2}{\alpha_1}}\frac{Z_s}{\sqrt{\zeta_{ss}}}\right] \tag{10}$$

Equations (9) and (10) constitute a nonlinear system of equations with two unknowns whose solutions can only be done on a digital computer. However, some simplifications are feasible at high Peclet numbers. First, Eq. (4b) is approximated as

$$\zeta(\tau,\theta) = (\sin^4\theta \cos^2\theta)\lambda \qquad \text{for } \lambda \le O(1) \tag{11a}$$

$$\zeta(\tau,\theta) = \frac{1}{4}\sin^4\theta\left\{\frac{(1-\sin^4\theta)}{(\cos^2\theta e^{-2Pe\tau}+\sin^2\theta)^2}\right\} \qquad \text{for } \lambda \ge O(10) \tag{11b}$$

For simplicity, $Pe\tau$ has been replaced by λ. Since expressions (11a) and (11b) are accurate to within 10%, they may be used reasonably to cover the whole range of λ. With these approximations, the total heat balance is found by integrating Eq. (10) over the surface of the liquid sphere. The results are

$$\int_0^\lambda \left(\frac{A_2}{\sqrt{\lambda-\lambda^*}} + A_1\right) Z_s(\lambda^*)\mathrm{d}\lambda^* + \int_0^\lambda X_s(\lambda^*)\mathrm{d}\lambda^* = 2A_2\sqrt{\lambda} \qquad \text{for } \lambda \le O(1) \tag{12a}$$

$$\int_0^\lambda \left\{A_2\left[1-\left(1-\frac{\pi}{2}\right)e^{-2(\lambda-\lambda^*)}\right]+A_1\right\}Z_s(\lambda^*)\mathrm{d}\lambda^*+\int_0^\lambda X_s(\lambda^*)\mathrm{d}\lambda^*$$

$$=A_2\left[\lambda+\left(\frac{1}{2}-\frac{\pi}{4}\right)\left(1-e^{-2\lambda}\right)\right] \qquad \text{for } \lambda\geq O(10) \qquad (12b)$$

where A_i is defined as

$$A_i=\frac{K_i(T_0-T_\infty)}{\rho_1\Delta H_v\sqrt{D\alpha_i}} \qquad (13)$$

Equation (12) is a special type of Volterra integral equation that can be solved with the Laplace transform. However, the exponential behavior of Eq. (9) makes such a powerful transform technique obsolete. To simplify the analysis, we consider the linear version of Eq. (10) in the form:

$$X_s=B+CZ_s \qquad (14)$$

where B and C are defined as

$$B=1+\frac{W_1}{W_2}\left\{1-P_\infty\left[1+\frac{\Delta H_v}{R_gT_b^2}(T_0-T_\infty)Z_b\right]\right\} \qquad (15a)$$

$$C=\frac{W_1}{W_2}\left(\frac{P_\infty\Delta H_v}{R_gT_b^2}\right)(T_0-T_\infty) \qquad (15b)$$

In writing Eq. (14), it has been assumed that the surface temperature is not substantially different from the normal boiling temperature of the liquid. With the addition of Eq. (14) for interfacial concentration, Eqs. (12a) and (12b) become:

$$\int_0^\lambda \left(\frac{A_2}{\sqrt{\lambda-\lambda^*}}+A_1+C\right) Z_s(\lambda^*)\mathrm{d}\lambda^*=2A_2\sqrt{\lambda}-B\lambda$$

$$\text{for } \lambda\leq O(1) \qquad (16a)$$

$$\int_0^\lambda \left\{A_2\left[1+\left(1-\frac{\pi}{2}\right)e^{-2(\lambda-\lambda^*)}\right]+A_1+C\right\}Z_s(\lambda^*)\mathrm{d}\lambda^*$$

$$=(A_2-B)\lambda+A_2\left(\frac{1}{2}-\frac{\pi}{4}\right)(1-e^{-2\lambda}) \qquad \text{for } \lambda\geq O(10) \qquad (16b)$$

The solution of the preceding equation is straightforward, and the detailed calculation need not be shown here. In brief, one could get a solution by first taking the Laplace transform of Eqs. (16) using the convolution theorem, then performing the inverse transform of the dependent quantity to have

$$Z_s=1-\frac{2\beta}{\sqrt{\pi}}\left\{\lambda^{\frac{1}{2}}-\frac{2}{3}\gamma^2\lambda^{\frac{3}{2}}-\frac{4}{15}\gamma^4\lambda^{\frac{5}{2}}+\ldots\right\} \quad \text{for } \lambda\leq O(1) \quad (17a)$$

$$Z_s=\frac{\beta}{\gamma}+\left(\omega-\frac{\beta}{\gamma}\right)e^{-\gamma\lambda} \qquad \text{for } \lambda\geq O(10) \quad (17b)$$

where β and γ represent $(A_1+B+C)/(\sqrt{\pi}A_2)$, and $(A_1+C)/(\sqrt{\pi}A_2)$, respectively, when λ is small, otherwise $\beta=2(A_2-B)/[A_1+(\pi/2)A_2+C]$, $\gamma=2(A_1+A_2+C)/[A_1+(\pi/2)A_2+C]$, and $\omega=[2A_2-(\pi/2)A_2-B]/[A_1+(\pi/2)A_2+C]$. The total mass transfer rate now may be obtained by integrating the local mass flux over the entire surface area of the droplet. From mass conservation principle, it must equal the mass loss of liquid due to evaporation

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}(\ln R^2)=\frac{-2}{\sqrt{\pi}}\left(\frac{\rho_1}{\rho_2}\right)\sqrt{(D/\alpha_1Pe)}\,(B+CZ_s) \qquad (18)$$

The ordinary differential equation (18) can be readily integrated to give the droplet size history

$$\ln\left(\frac{R}{R_0}\right)^2=\frac{-2}{\sqrt{\pi}}\left(\frac{\rho_1}{\rho_2}\right)\sqrt{(D/\alpha_1Pe)}\,\Big[(B+C)\lambda$$

$$-\frac{2\beta C}{\sqrt{\pi}}\left(\frac{2}{3}\lambda^{\frac{3}{2}}-\frac{4}{15}\gamma^2\lambda^{\frac{5}{2}}-\frac{8}{105}\gamma^4\lambda^{\frac{7}{2}}+\ldots\right)\Big] \qquad (19a)$$

$$\ln\left(\frac{R}{R_0}\right)^2=\frac{-2}{\sqrt{\pi}}\left(\frac{\rho_1}{\rho_2}\right)\sqrt{(D/\alpha_1Pe)}$$

$$\times\left\{B\lambda+C\left[\frac{\beta}{\gamma}\lambda+\frac{1}{\gamma}\left(\omega-\frac{\beta}{\gamma}\right)(1-e^{-\gamma\lambda})\right]\right\} \qquad (19b)$$

for short and long time, respectively.

Results and Discussion

Sample calculations were performed for an octane-air system at $Pe=200$, $T_0=380$ K, and $T_\infty=420$ K whose physical properties are chosen in accordance with the one-third rule. The results shown in Fig. 1 exhibit two distinct regions. During the early stage of the process, diffusion dominates the heat and mass transfer; hence, the motion induced by the electric field does not have any appreciable influence on the evaporation rates. As time elapses, the drastic change in the slope of the surface regression rate signifies that convection has established its dominance. Unlike diffusion, convection is much more efficient in that it is capable of bringing the warm and fresh fluid particles together for coalescence. As a result of the mixing mechanism, heat enters the drop interior at a rate much faster than that of diffusion alone, thereby shortening the internal heating time. Figure 2 shows the history of the dimensionless surface temperature Z_s as a function of time. It is seen that, with the small time solution, the surface temperature rises quickly to the vicinity of the boiling temperature. From the large time solution, the interfacial temperature is relatively insensitive to time and asymptotically approaches the boiling temperature. Based on the surface temperature plot, it is believed that the short and long time solutions are correct because only a very short extrapolation is needed to join the two results. It is noted that the droplet size plot in Fig. 1 is not very sensitive to the surface temperature variation in view of the very short transient nature of the temperature at the drop surface. Also, the fast time response of the surface temperature suggests that the formulation may be simplified somewhat further by removing the transience in the boundary conditions.

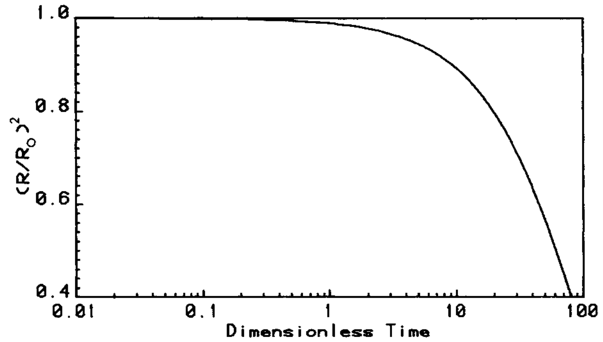

Fig. 1 Drop size history.

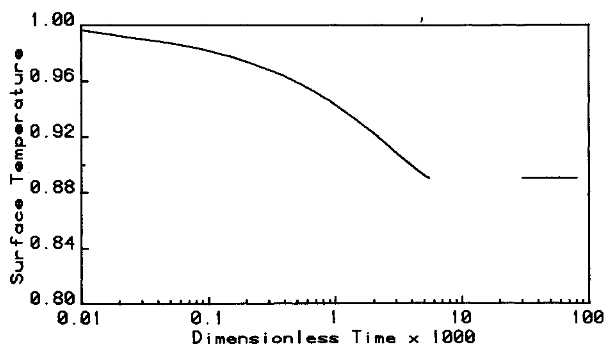

Fig. 2 Interfacial temperature vs time.

From the solutions for short and long time, it may be apparent that the enhancement of evaporation is proportional to $\sqrt{Pe}$, which, in turn, is proportional to the electric field strength. Although the results are encouraging, the current treatment is only applicable to the situations where the electric field generates a Taylor type of motion. At high field strength the strong current would create a magnetic field and a nonvanishing body force in the momentum equation. Such couplings and interactions must be accounted for in the analysis.

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Radiation View Factors from a Sphere to Nonintersecting Planar Surfaces

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Introduction

IN the calculation of radiative heat exchange between two bodies, the determination of a radiation view factor becomes essential. Although a large number of radiation shape factors for different configurations have been made available, due to the tedious task of quadruple integral, there still exist many important geometric configurations involving a sphere and a finite surface for which the associated view factors are not known.

The purpose of this study is to develop the radiation shape factors systematically from a sphere to a class of nonintersecting planes. Radiation between a sphere and a planar element was evaluated analytically by Chung and Sumitra[1] and Juul.[2] Feingold and Gupta[3] developed an analytical approach for the radiation view factors from spheres to various nonintersecting coaxial planes. Complementary to their work, Chung and Naraghi[4] developed closed-form solutions for the view factors from spheres to intersecting disks and to the external surface of a coaxial cylinder. They further developed a general formulation for the view factors between a sphere and a class of axisymmetrical bodies.[5] Balance and Donovan[6] studied the view factor between spheres and intersecting disks using a Monte Carlo method.

A logical continuation of the previous efforts would be the evaluation of the view factors from a sphere to any noncoaxial surfaces. To the best knowledge of these authors, the aforementioned case has not been solved.

Mathematical Formulation

Consideration is given to Fig. 1, which depicts the radiation from a sphere to a noncoaxial differential planar area; its plane does not intersect the sphere. The view factor from $\mathrm{d}A_1$ to A_2 has been found[4] as

$$F_{\mathrm{d}A_1-A_2}=\frac{\cos\theta}{(1+S)^2},\ \theta\le\cot^{-1}\left(\frac{r_s}{\sqrt{(s+r_s)^2-r_s^2}}\right) \tag{1}$$

where

$$S=\frac{s}{r_s}$$

Referring to Fig. 1, we have $\cos\theta=\mathrm{d}/\sqrt{x^2+y^2+\mathrm{d}^2}$ and $(1+S)^2=(x^2+y^2+\mathrm{d}^2)/r_s^2$. Substituting the expressions into Eq. (1), making use of the reciprocity rule, and carrying out the integration gives

$$F_{2-1}=\iint_{A_1}\mathrm{d}F_{A_2-\mathrm{d}A_1}=\frac{\mathrm{d}}{4\pi}\iint_{A_1}\frac{\mathrm{d}A_1}{(x^2+y^2+\mathrm{d}^2)^{3/2}} \tag{2}$$

If A_1 is symmetrical with respect to the x or y axis, Eq. (2) can be reduced to a single integration form. Suppose that A_1 is

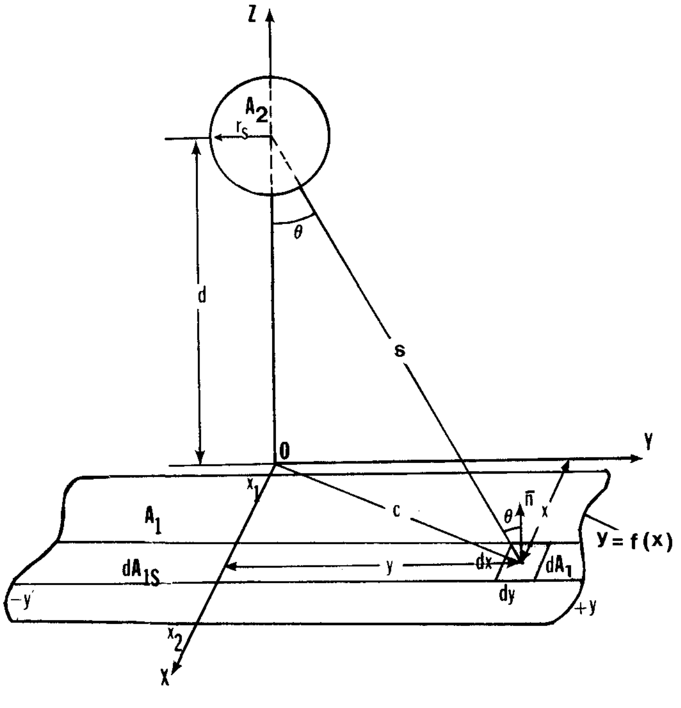

Fig. 1 Radiation between a sphere and a planar surface.

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